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Al-Fateh University, Faculty of Engineering
Electrical and Electronics Engineering Department

EE 303 Numerical Techniques and Programming
Midterm II, June 14th, 2009

a) Answer all the questions to the best of your knowledge.
b) Show all steps and carry all calculations up to 3 digits unless otherwise mentioned.
c) No question will be answered during the exam.
d) Time allowed: *2 hours*

Q1- Using the following data points

x_i	1.1	2.0	3.5	5.0	7.1
f_i	0.6981	1.4715	2.1287	2.0521	1.4480

P_1, P_2, P_3

(a) Use divide difference to find $f(1.75)$ with polynomials of degrees 1,2, and 3
(b) Given the true value $f(1.75)$= 1.27664, find the relative error in the three polynomials obtained in part (a) of this question.

1.5305

Q2-For the function $f(x) = 2x * \cos(2x)$, find $f'(0.2)$ using a forward-difference approximation, backward difference approximation and central-difference approximation using Δx=0.1,0.05 and 0.025. Show that the relative error is approximately halved when Δx is halved in forward and backward difference while the relative error is approximately quartered when Δx is halved in the central difference.

$f'(x) = 2x(-2\sin 2x) + 2(\cos 2x)$ at $x = 0.2$

Q3-
(a) Using 3rd degree Newton-Gregory forward interpolating polynomial that fits four evenly spaced point, derive Simpson's $\frac{3h}{8}$ formula.
(b) Use the formula obtained in part (a) of this question to find the following integral. ($h=0.2$)

$$\int_0^{1.2} \frac{dx}{(x^2+9)^3}$$

$\frac{x}{3}$ $\frac{12}{3}$

(c) Use the trapezoidal rule to find the same integral in part (b) of this question ($n=6$).
(d) Given the true value of the integral=0.001425, which method gave better approximation in term of relative error.

Good luck to all of you.

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Miley Cyrus

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Electrical and Electronics Engineering Department

EE 303 Numerical Techniques and Programming

Midterm I, November 25th, 2008

- Answer all questions to the best of your knowledge.
- Programmable calculators are not allowed
- No question will be answered during the exam.

Q1- Time allowed 90 *minutes*

(a) If the exact answer is A and the approximate answer is $\tilde{A}$, find the absolute and relative error when

1) A=10.147, $\tilde{A} = 10.159$
2) A=0.0047, $\tilde{A} = 0.0045$
3) A=0.671 x 10^{-12}, $\tilde{A} = 0.669 \times 10^{-12}$

(b) Find the Taylor series expansion of $f(x) = e^{x^2}$

(c) Find a recursive formula of the form T_n=(...) T_{n-1} for the function in part (b) of this question

(d) Write a c/c++ program to compute the series in part c of this question.

$\frac{12x^2}{(2*1)(2x)}$

Q2- Use Newton's method to find the intersection of the following two curves **(10 Marks)**

$$x^2 + 3y^2 - 1 = 0$$

$$\cancel{(x-2)^2 + (y-1)^2 - 4 = 0}$$

with x_0=0 and y_0=0.5, perform only 3 iterations.

$x^2 - 4x + 4 + y^2 - 2y + 1 - 4 = 0$

Q3- Solve the system of equations: **(10 Marks)**

$$x_1 - x_2 + 2x_3 = 4$$
$$2x_1 + x_2 + 5x_3 = 5$$
$$-x_1 + 4x_2 + x_3 = -7$$

$\frac{12x^2}{2*1} = (\quad) 2x$

(a) Using Gaussian elimination with partial pivoting

(b) Show that same answer can be obtained using crammer's rule

(10 Marks)

Good luck to all of you

$(e^x)^2$

$e^{2x} = e^{2x} \cdot 2 \quad \frac{d}{dx} e^x = e^x dx \quad e^{x^2} = f(0) + x f'(0) + \frac{x^2}{2!} f''(0)$

$= 1 + x(0) +$

$\frac{d}{dx} e^{x^2} = e^{x^2} \cdot 2x\,dx \quad \frac{d}{dx} e^{x^2} = e^{x^2} \cdot 2x\,dx$

$f'(x) = e^{x^2} \cdot 2 + 2x \cdot e^{x^2} 2x\,dx$

$2 + 0 \quad 4x^2 e^{x^2} dx \quad \cdot 8x$

$e^{x^2} \cdot 4x^2$ $f(0) = 2$

$f'(x) = 2e^{x^2} \cdot 2x\,dx + e^{x^2} \cdot 8x + 4x^2 \cdot e^{x^2} 2x\,dx$

$f''(x) = e^{x^2} \cdot 4 + 4x \cdot e^{x^2} 2x + e^{x^2} \cdot 8 + 8x \cdot e^{x^2} 2x + 4x^3 \cdot e^{x^2} \cdot 2x + e^{x^2} \cdot 8x$

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Midterm I, November 25th 2008

(a) Absolute True Error = Exact Error − Approximated Error

E_t = Exact ~~Value~~ Approximated ~~Value~~

Relative True Error = $\frac{\text{Absolute True Error}}{\text{Exact value}}$

$\in_t = \frac{E_t}{\text{Exact Value}}$

$\% \in_t = \frac{E_t}{\text{Exact value}} \times 100$

1) $A = 10.147$, $\bar{A} = 10.159 \Rightarrow E_t = 10.147 - 10.159$

$\therefore E_t = -0.012 \Rightarrow |E_t| = 0.012$

$\in_t = \frac{|E_t|}{10.147} = \frac{0.012}{10.147} \Rightarrow \in_t = 0.001183$

$\% \in_t = 0.1183 \%$

2) $A = 0.0047$, $\bar{A} = 0.0045$, work with absolute values

$E_t = |0.0047 - 0.0045| \Rightarrow E_t = 0.0002$

$\% \in_t = \frac{0.0002}{0.0047} \times 100 = \frac{0.2}{0.47} \times 100 \Rightarrow \in_t = 4.255 \%$

Even though the absolute error of (2) is less than that of (1) but the relative error of (2) is greater than that of (1)

$A = 0.671 \times 10^{-12}$, $\hat{A} = 0.669 \times 10^{-12}$

$E_t = |0.671 - 0.669| \times 10^{-12} = 0.002 \times 10^{-12}$

$\varepsilon_t = \frac{0.002 \times 10^{-12}}{0.671} \times 100 = 0.298$

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b) Find the Taylor series of $f(x) = e^{x^2}$

$\because f(x_{i+1}) = f(x_i) + f'(x_i).h + \frac{f''(x_i).h^2}{2!} + \frac{f'''(x_i).h^3}{3!} + \dots$

$= \sum_{n=0}^{\infty} \frac{f^{(n)}(x_i).h^n}{n!}$

$2x \quad 2x\, e^{x^2} + 2e^{x^2}$
$4x^2 e^{x^2}$

$f(x_{i+1}) = f(x_i) + \sum_{n=1}^{\infty} \frac{f^{(n)}(x_i).h^n}{n!}$

$\because f(x) = e^{x^2} \Rightarrow f'(x) = 2xe^{x^2}$, $f''(x) = 4x^2.e^{x^2}$

$f^{(3)}(x) = 8x^3.e^{x^2}$, $f^{(4)}(x) = 16x^4.e^{x^2}$

$\therefore f^{(n)}(x) = 2^n.x^n.e^{x^2} = (2x)^n.e^{x^2}$

$f^{(0)}(x) = e^{x^2}$

$f(x_{i+1}) = \sum_{n=0}^{\infty} \frac{(2x)^n.e^{x^2}.h^n}{n!}$ → If you want to use MATLAB

$= e^{x^2} + \sum \frac{(2x_i)^n.e^{x^2}.h^n}{n!}$ → using C program to avoid division by zero of factorial zero

c) $T_n = \frac{2x_i.T_{n-1}}{n}.h$

```
#include<stdio.h>
#    "   <math.h>
#    "   <conio.h>
main() {

int i=0, n;   // n is the number of terms
float xi, x, fofx, Er;   /* fofx is the value of f(xi+1)
float Tv, h;                Er is the relative True error */
                            /*Tv is the True value
                              h is step size */
Printf("\n Enter xi = "); Scanf(" %f", &xi);
Printf("\n Enter x = "); Scanf(" %f", &x);
Printf("\n Number of Terms = "); Scanf(" %d", &n);

h =fabs(x - xi);
Tv = exp(pow(x,2));     // calculate the True value

Printf("\n h = %.5f \n True value = %.5f ", h, Tv);

fofx = exp(pow(xi,2));
Er = 100*fabs((Tv- fofx)/Tv);

Printf("\n Iteration        f(x)        R.T.E.P ");
Printf("\n %d               %.5f        %.5f ", i, fofx, Er);
i=i+1;
while(i<=n)
{
fofx = (2*xi)*fofx*h/i ;

i=i+1;
}
getche();
```

$$\underbrace{x^2+3y^2-1}_{h(x,y)} = \underbrace{(x-2)^2+(y-1)-4}_{g(x,y)}$$

At the interseption point $h = g$

$$\therefore x^2+3y^2-1-(x-2)^2-(y-1)+4=0$$

$$x^2+3y^2-\cancel{1}-x^2+4x-4-y+\cancel{1}+4=0$$

$$3y^2-y+4x=0$$

Set $\boxed{f(x,y)=3y^2-y+4x}$

Function of Two Variables

$$\frac{\partial f}{\partial x}=4$$

i. guess it is out of the scope of the course

$$\frac{\partial f}{\partial y}=6y-1$$

$$x_{i+1}=x_i-\underline{\qquad\qquad}$$

Q3

$$x_1-x_2+2x_3=4$$
$$2x_1+x_2+5x_3=5$$
$$-x_1+4x_2+x_3=-7$$

a) $\begin{bmatrix} 1 & -1 & 2 & 4 \\ 2 & 1 & 5 & 5 \\ -1 & 4 & 1 & -7 \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 1 & 5 & 5 \\ 1 & -1 & 2 & 4 \\ -1 & 4 & 1 & -7 \end{bmatrix}$

~~eliminat~~. Forward elimination (backward substitution)

$$f_{12}=\frac{a_{12}}{a_{11}}=\frac{1}{2}=0.5$$

$$f_{13}=\frac{-1}{2}=-0.5$$

$R_2 - f_{12} \cdot R_1$ $\quad (1 - \frac{1}{2} \times 2) \quad -1 - \frac{1}{2} \times 1 \quad 2 - \frac{1}{2} \times 5 \quad 4 - \frac{1}{2} \times 5$

$= R_3 - f_{12} \cdot R_1$ $\quad -1-(-0.5)\times 2 \quad 4-(-\frac{1}{2}\times 1) \quad 1-(-\frac{1}{2}\times 5) \quad -7-(-\frac{1}{2}\times 5)$

$$\begin{bmatrix} 2 & 1 & 5 & 5 \\ 0 & -\frac{3}{2} & -\frac{1}{2} & \frac{3}{2} \\ 0 & \frac{9}{2} & \frac{7}{2} & -\frac{9}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} 2 & 1 & 5 & 5 \\ 0 & -1.5 & -0.5 & 1.5 \\ 0 & 4.5 & 3.5 & -4.5 \end{bmatrix}$$

a'_{32}

Eliminating x_2 from R_3

$$f_{23} = \frac{a_{32}}{a_{22}} = \frac{4.5}{-1.5} \Rightarrow f_{23} = -3$$

$0 \quad 0 \quad 3.5-(-3\times -0.5) \quad -4.5+3\times 1.5$

$$R_3 \leftarrow R_3 - f_{23} \cdot R_2 \Rightarrow \begin{bmatrix} 2 & 1 & 5 & 5 \\ 0 & -1.5 & -0.5 & 1.5 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

$\therefore 2x_3 = 0 \Rightarrow x_3 = 0$

$-1.5x_2 - 0.5\times 0 = 1.5 \Rightarrow x_2 = -1$

$2x_1 + 1(-1) + 5(0) = 5 \Rightarrow x_1 = 3$

Substitute in the original eq$_s$

$x_1 - x_2 + 2x_3 = 4 \Rightarrow \quad 3 - (-1) + 2(0) = 4$

$3 + 1 + 0 = 4$

$4 = 4$ $\#$

$2x_1 + x_2 + 5x_3 = 2(3) - 1 + 5(0) = 5 = L.H.S$

$-x_1 + 4x_2 + x_3 = -(3) + 4(-1) + 0 = -7 = L.H.S$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$$

$$\therefore \Delta = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & 5 \\ -1 & 4 & 1 \end{vmatrix} = 1(1-20)+1(2+5)+2(8+1)$$

$$\therefore \boxed{\Delta = 6}, \quad \Delta_x = \begin{vmatrix} 4 & -1 & 2 \\ 5 & 1 & 5 \\ -7 & 4 & 1 \end{vmatrix} = 4(1-20)+1(5+35)+2(20+7)$$

$$= 4(-19)+40+2(27)$$

$$= -76+40+54 = -36+54$$

$$= 18$$

$$\boxed{\Delta x_1 = 18}$$

$$\Delta_y = \begin{vmatrix} 1 & 4 & 2 \\ 2 & 5 & 5 \\ -1 & -7 & 1 \end{vmatrix} = 1(5+35)-4(2+5)+2(-14+5)$$

$$= 40 - 4\times 7 + 2(-9) = 40-28-18 = -6$$

$$\boxed{\Delta x_2 = -6}$$

$$\Delta x_3 = \begin{vmatrix} 1 & -1 & 4 \\ 2 & 1 & 5 \\ -1 & 4 & -7 \end{vmatrix} = 1(-7-20)+1(-14+5)+4(8+1)$$

$$= -27-9+36 = 0$$

$$\Delta x_3 = 0$$

$$x_1 = \frac{\Delta x_1}{\Delta}, \quad x_2 = \frac{\Delta x_2}{\Delta}, \quad x_3 = \frac{\Delta x_3}{\Delta}$$

$$x_1 = \frac{18}{6} = 3, \quad x_2 = \frac{-6}{6} = -1, \quad x_3 = \frac{0}{6} = 0$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$$

The same result as in (b).

Group B

Alfateh University

Electrical Engineering Department
EE303 Numerical Analysis
Mid-Term I

Answer all questions, Carry calculations to 3 decimal places, time allowed 1.5 hours

$T_{i+1} = \frac{x^2}{(n+2)(n\ldots}$

$T_i = \frac{x}{1!}$

Q1-

a) Write a recursive expression in the form of $T_{i+1} = (\ldots)\, T_i$ for the following expression

$$\cosh x = 1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \frac{1}{720}x^6 + \frac{1}{40{,}320}x^8 + \ldots$$

(3 Marks)

b) Starting at [57, 33] using the bisection method, what will be the relative error after 52 iterations.

(3 Marks)

c) Write a *C program* to perform bisection method for finding the root of nonlinear equation.

(4 Marks)

Q2 a) Using Newton's method, find the roots of the following two equations:

$f_1(x,y) = 1 + x^2 - y^2 + e^x\cos(y)$ and $f_2(x,y) = 2xy + e^x\sin(y)$. Start with $x_0 = 1.0$ and $y_0 = 4$

Perform 3 iteration

(7 Marks)

b) Use the secant method to find the root of the following equation:
$f(x) = 3x + \sin(x) - e^x$, repeat until the error drops to below 10^{-5}

(3 Marks)

Q3 a) Given $A = \begin{bmatrix} 4 & -2 & 1 \\ -3 & -1 & 4 \\ 1 & -1 & 3 \end{bmatrix}$, Find L and U of A and show that det(A)=det(L)*det(U)

$\boxed{LU}$

LU

(6 Marks)

$L^{-1}c = c$

b) Given $b = \begin{bmatrix} 15 \\ 8 \\ 13 \end{bmatrix}$, solve the system $Ax = b$ using Gaussian elimination method

(4 Marks)